

# A DERIVED ISOMETRY THEOREM FOR CONSTRUCTIBLE SHEAVES ON $\mathbb{R}$

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Persistent homology has been recently studied with the tools of sheaf theory in the derived setting by Kashiwara and Schapira [KS18a] after J. Curry has made the first link between persistent homology and sheaves.

We prove the isometry theorem in this derived setting, thus expressing the convolution distance of sheaves as a matching distance between combinatorial objects associated to them that we call graded barcodes. This allows to consider sheaf-theoretical constructions as combinatorial, stable topological descriptors of data, and generalizes the situation of persistence with one parameter.

On a second time, we relate sheaf-theoretic and persistence-theoretic constructions, and show how the derived isometry theorem allow to give a new, deeper, interpretation of level-set persistence stability.

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