INTEGRABLE BILLIARDS: GENERALIZATIONS AND APPLICATIONS TO MECHANICS

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Let us recall that billiard system describes motion of a particle in a flat domain Ω with piecewise smooth boundary P. Reflection should be elastic. The Hamiltonian is the square of velocity vector.

D.Birkhoff proved the integrability if P is an ellipse. V.V.Kozlov, D.V.Treschev proved that integrability preserves for P that consists of arcs of confocal ellipses and hyperbolas. This system has an additional first integral Λ which value is some parameter of the caustic for trajectory.

Fixing $|\overrightarrow{v}|^2 = h$ one have 3-dimensional manifold Q^3 foliated on level surfaces of Λ. Such foliations are smooth-wise analogs of Liouville foliations investigated by A.T. Fomenko school. Fomenko–Zieschang invariant (graph with numerical marks, vertices correspond to singularities of the foliation) classifies them in the sense of Liouville equivalence. Two integrable systems are called equivalent if piece-wise diffeomorphism exists. Their trajectory closures also have the same structure.

Fomenko–Zieschang invariant for billiards in flat domains were calculated by V. Dragovich, M. Radnovich and V.V. Vedyushkina (Fokicheva). Let us call such plane domains elementary domains.

Now we describe a generalization of such billiard. Let us glue together several elementary domains-sheets along common borders. Produced domain has a structure of CW-complex. Let us call it a *billiard book*. An interesting problem is to describe the Liouville foliation of the obtained billiards.

Previously, the case of only two glued domains-sheets was considered. Produced domains were called *topological billiards*. They were completely investigated in terms of the Fomenko-Zieschang invariant in [3]. Later these invariants were calculated for wide class of non-trivial billiard books.

On the other hand Fomenko-Zieschang invariants were calculated for many integrable cases of the rigid body dynamics and geodesic flows. It allows to detect the Liouville equivalence of these systems to some topological billiards by comparing the invariants (see [2]). It means that billiard books and topological billiards "visually model" many fairly complicated integrable cases in the dynamics of the rigid body. This simulation makes it possible to present and effectively classify the stable and unstable periodic trajectories of integrable systems, in particular, in physics and mechanics.

For example, the Euler case can be simulated by the billiards for all values of energy integral. Such billiard simulation is done for the systems of the Lagrange top and Kovalevskaya top, then for the Zhukovskii gyrostat, for the systems by Goryachev-Chaplygin -Sretenskii, Clebsch, Sokolov, the Kovalevskaya-Yahia case (Kovalevskaya top with gyrostat) for many values of energy.

Also it was possible to apply results of our calculation to modeling integrable geodesic flows on orientable 2-dimensional surfaces. Namely, all such flows that have linear or quadratic first integral were modeled by integrable billiards. It means that every linear or quadratic integral can be realized in this sense by one, canonical Hamiltonian and quadratic first integral.

References

[1] A. T. Fomenko and A. V. Bolsinov: *Integrable Hamiltonian Systems: Geometry, Topology, Classification*, CRC Press, 2004.

[2] V. V. Vedyushkina, A. T. Fomenko: *Integrable topological billiards and equivalent dynamical systems*, Izv. Math., **81**, No. 4 (2017), 688733

[3] V. V. Fokicheva (Vedushkina): *A topological classification of billiards in locally planar domains bounded by arcs of confocal quadrics*, Sb. Math., **206** No. 10 (2015), 1463-1507.

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