REFINING PERSISTENCE VIA ENRICHED TOPOLOGICAL SUMMARIES

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Persistent Homology can be thought of as a mapping from various spaces of data to the stratified space of persistence diagrams. By considering inverse problems for the persistence map, we are led to various enrichments of classical tools in TDA: chiral, and decorated, merge trees; Reeb graphs equipped with a (nesting) poset structure on their fibers, and braided Reeb graphs. Some of these enriched topological summaries serve a two-fold purpose: 1) they provide discrete combinatorial structures that are in bijection with certain equivalence class structures put on the fiber of the persistence map, and 2) they are themselves a computationally efficient augmentation of persistence with sharper distinguishing power than persistence alone.

As an illustration of this first purpose I will show an extension of the formula in [Curry 2017] for counting "height equivalence" classes of functions on the two-sphere. The class of functions considered here (the "space of data types" mentioned above) are restricted to height functions on the two-sphere gotten by embedding the two-sphere in $\mathbb{R}^3$ and projecting onto a fixed direction vector $v \in S^{d-1}$. Height equivalence of functions is given by considering embedded spheres up to level-set preserving isotopy. Such an equivalence relation provides a discretization of the fiber of the persistent homology *transform* (PHT) evaluated at the single direction v. In the case where the function $h_v(x) = x \cdot v$ has only N minima, $N-1$ saddles, and 1 global maximum, the formulas in [Curry 2017] generalize directly, but chirality becomes replaced with a "nesting" poset structure on the fibers of the Reeb graph. However, for embeddings that yield height functions with two or more maxima, one has to consider braided Reeb graphs that respect the nesting relationships on the fibers. If time allows it, the injectivity results of [CMT 2018] and the characterizations of non-injectivity described above will be related and further questions will be posed.

References

[Curry 2017] Curry, J. "The Fiber of the Persistence Map" arXiv e-prints (currently accepted with minor revisions at JACT), June 2017, arXiv:1706.06059

[CMT 2018] Curry, J., Mukherjee, S., & Turner, K. 2018, arXiv:1805.09782

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