CYCLES IN RANDOM SIMPLICIAL COMPLEXES, LARGE AND SMALL

MATTHEW KAHLE

The topology of random simplicial complexes has been studied intensely for the past fifteen years or so. Some of the main topics that have been studied include: when is homology vanishing or non-vanishing, if non-vanishing how large are the Betti numbers, etc. See Chapter 22 of [1] for a recent survey.

In this talk, we are interested in a slightly more refined and geometric picture—how large are the cycles in a random complex? Of course the answer depends on how we measure the sizes of cycles and also on the model of random complex.

We will see that for the Linial–Meshulam random 2-complex, most cycles are large [2]. This inspires a proof of the existence 2-dimensional simplicial complexes with nearly optimally large homological systoles. This proof depends on the probabilistic method, and at the moment we have no idea how to construct such complexes explicitly.

On the other hand, we will also see that for a random geometric complex, all the cycles are small [3]. Here we measure the size of holes in terms of persistent homology. We show that the maximally persistent cycles are sub-logarithmic in size. This work is inspired by questions in topological data analysis, trying to separate topological signal from noise.

We will define these models of random complex as we go, and the talk will aim to be self contained.

REFERENCES

[1] Handbook of discrete and computational geometry. Third edition. Edited by Jacob E. Goodman, Joseph O'Rourke and Csaba D. Tóth. Discrete Mathematics and its Applications (Boca Raton). CRC Press, Boca Raton, FL, 2018. xxi+1927 pp. ISBN: 978-1-4987-1139-5

[2] Dotterrer, Dominic; Guth, Larry; Kahle, Matthew. 2-complexes with large 2-girth. *Discrete Comput. Geom.* 59 (2018), no. 2, 383–412.

[3] Bobrowski, Omer; Kahle, Matthew; Skraba, Primoz. Maximally persistent cycles in random geometric complexes. *Ann. Appl. Probab.* 27 (2017), no. 4, 2032–2060.

(Matthew Kahle) OHIO STATE UNIVERSITY
Email address: mkahle@math.osu.edu

NSF-DMS #1352386.