CLASSIFICATION OF 2D HAMILTONIAN VECTOR FIELDS AND TOPOLOGICAL FLOW DATA ANALYSIS: THEORY, COMPUTATION AND APPLICATIONS

TAKASHI SAKAJO

Fluid dynamics has been one of the important subjects of science and technologies. Numerical simulations of fluid equations play a significant role in the developments of modern infrastructure such as cars, high-speed trains, airplanes and wind turbine generators. Owing to the recent improvements of observation and measurement technologies, it is also utilized to extract useful information from ultrasonic images of cardiovascular flows and satellite images of ocean and coastal flows. On the other hand, although a large amount of visualized flow data is available, it is sometimes very difficult to express those flow patterns in words. The lack of common language among researches in multiple disciplines gives rise to an obstacle to proceed the interdisciplinary research. Moreover, with the explosion of data size obtained by such numerical simulations, observations and measurements, it is strongly desired to develop an efficient way describing flow properties and making their predictions from those massive data. To respond to these demands, we have developed a new classification theory for global streamline patterns of two-dimensional incompressible flows by making use of topology, discrete mathematics and the theory of dynamical systems.

What we have developed is a combinatorial classification for structurally stable Hamiltonian vector fields on multiply connected planar domains in the presence of a uniform flow, which is a model of two-dimensional incompressible fluid flows. The theory allows us to assign a unique sequence of letters, called *maximal words* and *regular expressions*, to every global topological structure created by the Hamiltonian vector fields [1, 4]. See the schematic of Figure 1 explaining the basic idea how the sequence of letters is assigned to streamline topologies. The conversion to maximal words and regular expressions is easy to implement, and the sequence of letters are intuitively interpretable to those who are not familiar with mathematics. An automatic conversion algorithm has already been implemented on computers as a software, and it is thus applicable to massive flow pattern data obtained by numerical simulations and/or physical measurements in fluid science, engineering and medical studies. By extracting global topological information from flow data, one is expected to figure out latent knowledge that are not recognized by experts in those fields so far. For instance, as demonstrated in [3], a certain flow functionality such as the maximum/minimum drag-to-lift ratios acting on a wing in the presence of a uniform flow is encoded as a specific sequence of letters contained commonly in maximal words and regular expressions of data-sets, which means that the sequence of letters works as a "DNA" for flows. See the article [5] for the underlying concepts in our theory. In addition, we have also developed a mathematical theory describing all possible global transitions of streamline topologies, without exceptions, through marginal structurally unstable Hamiltonian vector fields in terms of the changes of the sequence of letters [2]. Hence, by simply comparing them, we predict the change of global flow patterns that could possibly happen in future.

We will also introduce a new way of topological data analysis, called *topological flow data analysis (TFDA)*, based on the classification theory. Owing to TFDA, long-time evolutions of flows (or Hamiltonian vector fields), whose data size often exceeds more than giga-bytes, is drastically compressed into a small size of text data expressing the change of streamline topologies, which is amenable to statistical and/or time-series analysis, and machine learning for global topological information with ease. We show some applications to medical images of cardiovascular flows and flow patterns in meteorology. We also show another example illustrating that TFDA is available to create a data-driven model predicting a complex flow phenomenon.

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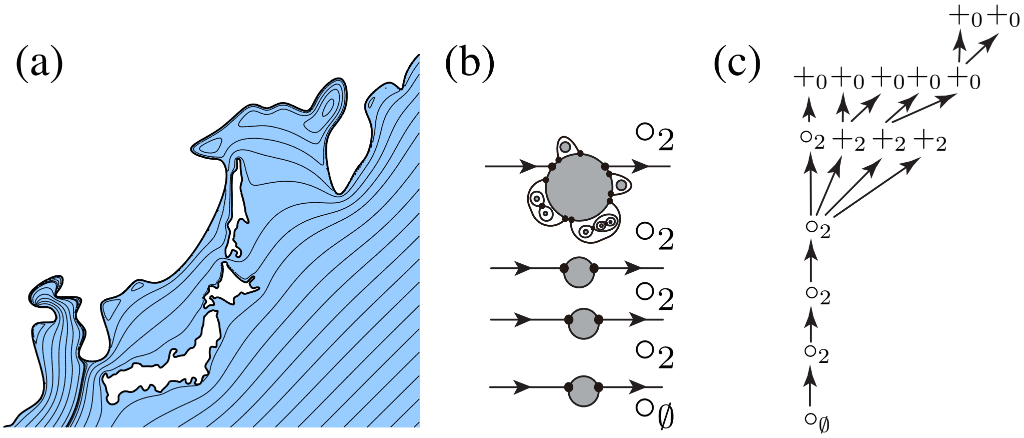

FIGURE 1. Encoding the maximal word and the regular expression to a numerically constructed streamline pattern around East-Asia ocean current shown in (a). Extracting topological pattern structures from the streamlines and assigning specific symbols to each domains separated by those topological streamlines as in (b), we then construct a tree structure to the adjacent relations between those domains as we see in (c). The tree structure is expressed as the unique regular expression $\circ_\emptyset(\circ_2(\circ_2(\circ_2(\circ_2(-_2), +_2(+_0, +_0), +_2(+_0, +_0(+_0, +_0)), +_2)))$. The maximal word for this streamline topology is given by $IA_2A_2A_2CCCCB_0B_0B_0$ according to [1, 4]. This process is now automatically executable with our computer software.

In the first part of my talk, starting with a brief review of the potential flow theory for those who are unfamiliar with fluid dynamics, we give the mathematics of the classification theory for structurally stable Hamiltonian vector fields. In the second part of my talk, we will explain how to implement the conversion algorithm as a computer software, and will introduce some applications of the theory to some flow problems and discuss future extensions of the theory. The contents of this talk is based on the joint works with Dr. T. Yokoyama (Kyoto University of Education), Dr. N. Nakano (Kyoto University) and Dr. T. Uda (Tohoku University).

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(Takashi Sakajo)
Email address: `sakajo@math.kyoto-u.ac.jp`