FOMENKO–ZIESCHANG INVARIANTS AND TOPOLOGY OF KOVALEVSKAYA INTEGRABLE SYSTEMS

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This talk will be devoted to topological invariants that classify foliations of integrable Hamiltonian systems [1]. They will be applied to describe closures of trajectories and topology of invariant submanifolds for integrable analogs of Kovalevskaya case in rigid body dynamics.

Let us recall that a system $v = \operatorname{sgrad} H$ with 2 degrees of freedom is integrable (in Liouville sense) if it has a first integral K that is independent of the Hamiltonian H (the energy integral). The phase space is foliated on 2-dimensional tori (Liouville theorem) and some *special fibers* that contain all *critical points*, i.e. points where the momentum map (H, K) has $\operatorname{rk} < 2$.

Critical points of (H, K) are not isolated in *regular* $Q^3_h = \{x \mid H(x) = h\}$ ($\operatorname{grad} H \neq 0$ in Q^3). They are united in several critical S^1, i.e. closed orbit of $v = \operatorname{sgrad} H$. Every critical orbit of v belongs to a special fiber. Recall that a function F on Q^3 satisfies *Morse–Bott condition* (is a *Bott function*) if its restriction f is a Morse function on transversal section to every closed critical orbit S^1 of v in Q^3_h. If first integral K is a Bott function on Q^3 then one can describe trajectories of the system on this level of energy in terms of their 2-dimensional closures and their bifurcations through critical fibers.

Closure of almost every trajectory is a Liouville torus. Fiber-wise neighbourhoods of special fibers were effectively classified by A. Fomenko (classes were called "3-atoms"). They have structure of S^1-fibration. A. Oshemkov classified bases of 3-atoms (called 2-atoms) using so-called f-graphs [2].

The next step was done by A. Fomenko and H. Zieschang [3]. They constructed graph invariant with some labels ("molecule") that classifies Liouville foliations on Q^3. Two manifolds are fiber-wise diffeomorphic iff invariants of systems coincide. Closures of trajectories also have the same structure.

These invariants were calculated for various mechanical and physical systems by many authors. Famous Euler, Lagrange, Kovalevskaya [4] cases of integrability in rigid body dynamics, Klebsh and Steklov integrable cases for a body motion in liquid and new cases of integrability (Sokolov and Bogoyavlenskii cases) are among them. Moreover, some technic (expression of some bases of $H^1(T^2)$ via so-called λ-cycles) helps use this theory for every 3-dimensional fiber-wise submanifold Q^3 of a symplectic manifold M^4 (not only for isoenergy submanifolds Q^3_h).

Some of these cases, i.e. Kovalevskaya and Sokolov, have integrable analogs on orbits of coadjoint representation in the dual space of the Lie algebras $so(3, 1)$ and $so(4)$ [5]. We will present these invariants for Kovalevskaya cases on $so(3, 1)$ and $so(4)$. Topological type (class of diffeomorphisms) of isoenergy submanifolds in the case of $so(4)$ also will be discussed: they were determined without any numerical calculations, only by analysing Fomenko–Zieschang invariants.

References

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