PERSISTENCE CURVES: A NEW VECTORIZATION OF PERSISTENCE DIAGRAMS

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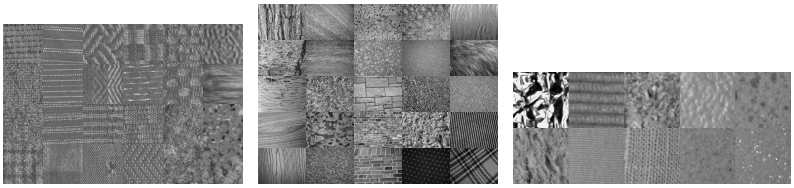
Persistence diagrams are a main tool in the field of Topological Data Analysis (TDA). They contain fruitful information about shapes of underlying objects. However, performing machine learning algorithms or statistical methods directly on persistence diagrams is a challenging problem due to the limitation of the space of persistence diagrams. For that reason, summarizing and vectorizing these diagrams is an important topic currently researched in TDA ([2, 1]). In this work, we develop a new way of summarizing diagrams: *Persistence Curves* (PC), and show practical uses of PC to several texture datasets.

The first part of the work devote to the foundation and theory of PCs. The main construction of PCs comes from the Fundamental Lemma of Persistent Homology, which reveals Betti numbers from persistence diagrams. As an example, Euler Characteristics Curve (ECC) is a special case of PC. PCs are family of curves and hence they can be used in a variety of situations depending on the data. We prove a rigorous bound for a general family of PCs. In particular, certain family of PCs admit the stability property. Furthermore, we show that Persistence Landscapes (PL) are special cases of PCs. PC provides the bridge from the classical ECC to modern PL.

The second part of the work is to apply PCs to real world applications. We investigate classifications of texture images on the three well-know texture datasets: Outex [4], UIUCTex [5], and KHT-TIP [7], where sample images are shown in Figure 1. Our results outperform some of TDA methods [3, 6] that applied to Outex. The performances for UIUCtex and KTH also reveal strong evidence. PCs are intrinsic characteristics of textures. Finally, we will show that PCs are simple and intuitive to implement.



(A) Outex (B) UIUCtex (C) KTH-Tips

FIGURE 1. Snapshots of the texture databases. Our best classification rate for each database are 99%, 92.4%, and 91.5%, respectively.

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