PERSISTENT HOMOLOGY OF RANDOM ČECH COMPLEXES ON MANIFOLDS

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The emerging research area known as random topology, motivated by many issues in manifold learning and Topological Data Analysis (TDA), comprises theoretical results that characterize the asymptotic behavior of topological properties of random objects. One aspect of this area is the study of random geometric complexes and their topological properties called Betti numbers and persistent Betti numbers. In this work, we concentrated on a typical type of random geometric complex, known as random Čech complex, denoted by $C(\mathcal{X}_n, r_n)$, if constructed on a finite set of points $\mathcal{X}_n = \{x_1, x_2, \dots, x_n\} \in \mathbb{R}^d$ and the radius is $r_n > 0$. Here, $\{r_n\}$ is a non-random sequence of positive numbers tending to zero for which three regimes (sparse regime, thermodynamic regime and dense regime) are divided according to the limit of $\{n^{1/m} r_n\}$: zero, finite or infinite, where $m \leq d$ is the intrinsic dimension of the space. It is known that the limiting behavior of Betti numbers in each regime is totally different.

We establish the strong law of large numbers for Betti numbers of random Čech complexes built on $\mathbb{R}^N$-valued binomial point processes in the thermodynamic regime [1]. Here we consider the case where the underlying distribution of the point processes is supported on a C^1 m-dimensional compact manifold embedded in $\mathbb{R}^d$. This result is new since only lower and upper bounds for the expectation of Betti numbers were known in the thermodynamic regime[3]. Moreover, from the applications point of view especially in TDA, considering only homology is not enough. It is important to see how persistent the 'holes' are, which constitutes the theory of persistent homology. We also extend our result for Betti numbers to persistent Betti numbers, and hence to persistence diagrams due to [2]. Here persistence diagram is regarded as a counting measure rather than as a muliset. All these results are proved under very mild assumption which only requires that the common probability density function belongs to L^p spaces, for all $1 \leq p < \infty$.

References

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