HARMONIC CYCLES AND RATIONAL WINDING NUMBERS

YOUNNG-JIN KIM AND WOONG KOOK

In this presentation, we discuss high-dimensional harmonic cycles. A harmonic cycle λ is a discrete harmonic form, i.e., a solution of the Laplacian equation

$$\triangle_n \lambda = 0 \tag{0.1}$$

with the Laplacian operator

$$\triangle_n = \partial_{n+1}\partial_{n+1}^t + \partial_n^t \partial_n \tag{0.2}$$

obtained from the chain complex $\partial_i : C_i(X) \to C_{i-1}(X)$ of a cell complex X. By the combinatorial Hodge theory, harmonic spaces are isomorphic to the homology groups with real coefficients. In particular, an acyclic cell complex has only the trivial harmonic cycle. In our talk, we will mainly address the case

$$\operatorname{rk} \widetilde{H}_n(X) = 1,\ \operatorname{rk} \widetilde{H}_{n-1}(X) = 0 \text{ and } \operatorname{rk} \widetilde{H}_{n+1}(X) = 0, \tag{0.3}$$

and introduce a formula for the *standard harmonic cycle* λ as a generator of the harmonic space,

$$\lambda = \sum_Y w(C_Y) C_Y \tag{0.4}$$

where the summation is over the cycletrees Y with its minimal cycle C_Y, and $w(\cdot)$ is the winding number map. We will also discuss intriguing combinatorial properties of λ with respect to (dual) spanning trees, (dual) cycletrees, winding number $w(\cdot)$ and cutting number $c(\cdot)$, i.e., for example,

$$\lambda \circ z = k_n(X) w(z) \text{ and } \lambda \circ z = k^n(X) c(z) \tag{0.5}$$

where $k_n(X)$ is the n-th tree number and $k^n(X)$ is the n-th dual tree number.

Furthermore, we will present an application for detecting the oscillation in flows for a periodic dynamical system with random perturbations through a simple example.

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(Kim, Y. J.) DEPARTMENT OF MATHEMATICAL SCIENCES, SEOUL NATIONAL UNIVERSITY, SEOUL 08826, SOUTH KOREA

Email address: sptz@snu.ac.kr

(Kook, W.) DEPARTMENT OF MATHEMATICAL SCIENCES, SEOUL NATIONAL UNIVERSITY, SEOUL 08826, SOUTH KOREA

Email address: woongkook@snu.ac.kr

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