SECTIONAL CATEGORY À LA QUILLEN

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The Lusternik-Schnirelmann category of a space is a particular case of a more general invariant of maps, introduced by Schwarz [7], called the *sectional category*:

$$cat(X) = \text{secat}(* \hookrightarrow X).$$

Farber's Topological complexity [3] is also a particular case of sectional category, namely, it is the sectional category of the diagonal inclusion:

$$\text{TC}(X) = \text{secat}(X \hookrightarrow X \times X).$$

A *rational* space is a topological space whose homotopy groups are vector spaces over the rational numbers. To any nilpotent space X we can assign its *rationalisation map*, $\rho : X \to X_0$, where X_0 is a rational space and $\pi_*(\rho) \otimes \mathbb{Q}$ (or equivalently $H(\rho, \mathbb{Q})$) is an isomorphism. We can think of X_0 as a space capturing the *torsion free* information of X.

In both Sullivan's [8] and Quillen's [6] approach a functor $F\colon \mathbf{Top} \to \mathcal{A}$ is constructed, being $\mathcal{A}$ the category of commutative differential graded algebras or differential graded Lie algebras, respectively. These functors restricted to the category of finite type rational spaces turn out to be equivalences of homotopy categories. This means that the rational (torsion free) homotopy type of X is completely encoded algebraically in $F(X)$!

This is very useful because it permits us to study any rational homotopy invariant in purely algebraic terms. We therefore speak of $F(X)$ (and any object equivalent to it) as a *model* for X. In particular, algebraic methods for computing invariants of the type of topological complexity can be developed. An example of this is the main result of [2] where we give a purely algebraic characterisation of sectional category.

The study of sectional category for rational spaces has been done using only Sullivan minimal models. The reason for this is that they are ideal objects for modelling products and fibrations: the tensor product of minimal models is the model of the cartesian product. For Quillen models the situation is much more difficult. In [9], D. Tanré gave a way of constructing the minimal Quillen model for the product of spaces, where the construction of the differential is not explicit. Later on, G. Lupton and S. Smith gave an explicit differential for this model in the case that one of the factors is a co-h-space[5].

In our talk, we will develop techniques to study sectional category using Quillen models. For this it is crucial to find explicit Quillen models for products and diagonal inclusions.

This is done through the *infinity Quillen functor* introduced in [1]. This functor assigns to a commutative differential graded algebra (cdga) model of a space the minimal Quillen model of the space. The construction consists on dualizing the cdga model for the space to make a co-commutative differential graded coalgebra, then transfer this structure through a retract to get an C-infinity coalgebra structure on the rational homology of the space, which translates into an explicit differential of the

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minimal Quillen model.

Then we give a characterisation of LS category and sectional category through Quillen models using the Whitehead characterisation and a model for the fat-wedge.

We will outline possible applications to some open problems for rational sectional category. For instance, the relation between the sectional category of a map and the LS category of its homotopy cofibre [4] or the Ganea conjecture for rational topological complexity.

Lastly we will expose the computational tools that we have implemented in order to carry out tedious computations.

References

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