SIMPLICIAL KIRCHHOFF INDEX

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We introduce a high-dimensional analogue of Kirchhoff index which is also known as total effective resistance. This analogue, which we call simplicial Kirchhoff index $Kf(X)$, is defined to be the sum of simplicial effective resistances of all $(d + 1)$-subsets of the vertex set of a simplicial complex X of dimension d. For a d-dimensional simplicial complex X with n vertices, we give formulas for simplicial Kirchhoff index in terms of the pseudo inverse of the Laplacian L in dimension d and its eigenvalues:

$$Kf(X) = n \cdot \operatorname{tr} L^+ = n \cdot \sum_{\lambda \in \Lambda_+} \frac{1}{\lambda}$$

where L^+ is the pseudo-inverse of L, and Λ_+ is the set of non-zero eigenvalues of L. Using this formula, we obtain an inequality for a high-dimensional analogue of algebraic connectivity and Kirchhoff index, and propose these quantities as measures of robustness of simplicial complexes. In addition, we derive its integral formula and relate this index to a simplicial dynamical system.

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